Chaos, Solitons and Fractals 131 (2020) 109494

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

The effect of similarity on the evolution of fairness in the ultimatum game

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ARTICLE INFO

Article history:
Received 13 June 2019
Revised 25 September 2019
Accepted 14 October 2019
Available online 19 November 2019

Keywords:
Ultimatum game
Evolutionary games
Similarity
Fairness

ABSTRACT

The ultimatum game (UG) is a useful game model for investigating the evolution of fairness. In this paper, considering the similarity between individuals, we introduce a similarity parameter into the spatial UG and focus on the evolution of the average offer and acceptance threshold. Under this mechanism, individuals can be either more generous or stingier to those who they are similar to. The simulation result shows that the fairness of the system decreases when the strategy is affected by the similarity between players. The greater the influence of similarity, the more fairness is decreased. Equal treatment, hence, is the best way to obtain fairness. Our results may provide some critical insights into the effect of similarity and favoritism on fairness among people.

1. Introduction

The emergence of fairness among human behaviors is still an unsolved puzzle [1] and a subject that has been studied for years. One useful model addressing this mystery is the ultimatum game (UG) [2–4]. In the UG, two players are asked to divide a certain amount of money. One player acts as the proposer, who presents an offer of how to split the money; the other player acts as the responder, who decides whether to accept the offer. If the responder chooses to accept the offer, then the money shall be shared as proposed. Otherwise, neither player gets any money. Obviously, for rational players, any offer other than zero is better to accept than nothing. Therefore, the optimal strategy of this game should be to offer as little as possible as the proposer and accept any nonzero offer as the responder. However, human experiments on the UG have shown a very different outcome. It has been found that most people are willing to propose a fair share, while nearly half of the responders reject offers that are less than 30 percent of the total sum [5–8]. Such irrational outcomes have drawn the attention of game theorists in years of studies.

Many studies have investigated the fairness that arose in the UG. The pioneering work of Page and Nowak investigated the effect of spatial structure on fairness in the UG. It was found that when interactions are made within certain neighbors instead of all populations, a much fairer outcome is achieved, indicating that spatial structure has a significant effect on fairness in an evolutionary UG [9]. Inspired by this finding, a series of works were conducted on different network structures, such as regular networks [10,11], complex networks [12–15] and dynamic networks [16–21]. Other important indicators, such as select rate [22,23], stake size [24,25], strategy discreteness [26,27], imitation accuracy [28] and heterogeneous distribution [29–31], have also been proven to carry influential weight on a spatial evolutionary UG.

In most previous studies, a player's strategy (the offer p, and the acceptance threshold q) to different neighbors was always determined to be a fixed value. However, there have also been works that focused on fairness when strategy was affected by certain factors between different neighbors, such as social attributes. In fact, many works have considered social attributes in the study of game theory [32–43]. Capraro and Perc studied the mathematical models for studying certain aspects of moral behavior [44]. Nowak et al. developed a model that allowed proposers to adjust their p according to information from previous interactions and considered reputation as an influential factor when making offers [45]. Zhang and Fu proposed an intervention mechanism. They showed the effect of spitefulness and altruism on the p and q of

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https://doi.org/10.1016/j.chaos.2019.109494
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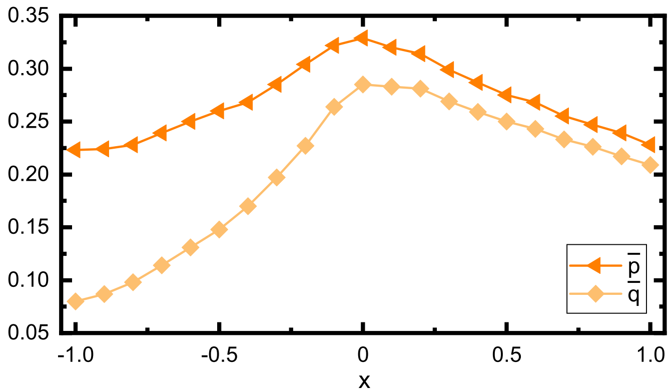

Fig. 1. (Color on-line) Average offer ($\bar{p}$) and acceptance threshold ($\bar{q}$) as a function of x, where x denotes the degree of how much similarity affects the game. Parameters: MCS = 10^6, N = 100*100, ε = 0.001.

different players, demonstrating that the above two interventions have opposite effects on the evolution of fairness in the UG [46].

In addition to reputation and spitefulness, the similarity between people can also play a significant role in real-life interactions. For example, studies have shown that people who are from the same social class are often more similar in aspects such as the generosity toward others [47] or the willing to take risks [48]. In this paper, we consider a similarity parameter in the standard spatial evolutionary UG. We assume that individuals can be either more generous or stingier to those whom they are similar to. In these two different situations, p and q between a pair of players are proportionally adjusted according to how similar they are. Under such a mechanism, fairness among a spatial network has presented a quite different evolution process, and some interesting facts have been found in our research.

2. Model

Our model is based on a 100*100 square lattice network with periodic boundary conditions, where each node is occupied by a player. Initially, each player i is randomly assigned with three values, the offer when acting as proposer, p_i $(0 < p_i < 1)$, the acceptance threshold when acting as responder, q_i $(0 < q_i < 1)$, and the social attribute denoting similarity, s_i $(0 < s_i < 1)$. The similarity between players i and j is defined as $\Delta s = |s_i - s_j|$ $(0 \le \Delta s < 1)$.

Each player plays with its von Neumann neighbors as both proposer and responder. With the effect of similarity, the specific values of p and q in a game where player i proposes an offer to player j are set as follows:

$$p_{ij} = min\left(p_i \cdot \frac{2}{1 + e^{-\Delta s \cdot x}}, 1\right) \tag{1}$$

$$q_{ij} = min\left(q_j \cdot \frac{2}{1 + e^{\Delta s \cdot x}}, 1\right), \tag{2}$$

where $x \in [-1, 1]$ is the parameter controlling the extent of similarity. With such settings, strategies can be adjusted according to the similarity between players within a reasonable range. When $0 < x \le 1$, we can obtain $p_{ij} > p_i$ and $q_{ij} < q_j$, indicating that players tend to be more generous and tolerant to similar others; when $-1 \le x < 0$, we can obtain $p_{ij} < p_i$ and $q_{ij} > q_j$, indicating that players tend to be stingier and stricter to similar others. When x=0, we can obtain $p_{ij} = p_i$ and $q_{ij} = q_j$, that is, a standard UG where similarity has no effect. Consequently, the payoff P that player i obtains is given by:

$$P_i = \sum_{j \in \Omega_i} P_{ij} = \sum_{j \in \Omega_i} \begin{cases} (1 - p_{ij}) + p_{ji}, & \text{if} \quad p_{ij} \ge q_{ji} \quad \text{and} \quad p_{ji} \ge q_{ij} \\ (1 - p_{ij}), & \text{if} \quad p_{ij} \ge q_{ji} \quad \text{and} \quad p_{ji} < q_{ij} \\ p_{ji}, & \text{if} \quad p_{ij} < q_{ji} \quad \text{and} \quad p_{ji} \ge q_{ij} \\ 0, & \text{if} \quad p_{ij} < q_{ji} \quad \text{and} \quad p_{ji} < q_{ij} \end{cases} \tag{3}$$

where Ω represents the von Neumann neighbors of node i.

Each player then chooses a site from its neighborhood (including itself) with the probability proportional to their cumulative payoff to inherit its (p, q) synchronously in the next time step with a small mutation [9]. If player i chooses player j for inheritance, then

$$(p_i(t+1), q_i(t+1)) = (p_j(t) + \delta_p, q_j(t) + \delta_q), \tag{4}$$

where δ is a random number from the interval: $(-\varepsilon, +\varepsilon)$ [14,20].

Each result used in our discussion is obtained by time average sampled at 10^4 interval from the $10^5 th$ generation to the $10^6 th$ generation [9], during which the system has reached equilibrium. We conduct 50 independent simulations and calculate the mean value for each parameter setting.

3. Result and discussion

In this paper, we focus on how the similarity between players affects the evolution of fairness. First, the results of $\bar{p}$ and $\bar{q}$ with different x values are shown in Fig. 1. Obviously, the variation in $\bar{p}$ and $\bar{q}$ are not monotonous with increasing x. When players tend to be more generous $(x > 0)$ or stingier $(x < 0)$ to others due to the effect of similarity, the fairness of the system weakens consequently. Equal treatment between players $(x = 0)$ is hence the best strategy to obtain fairness.

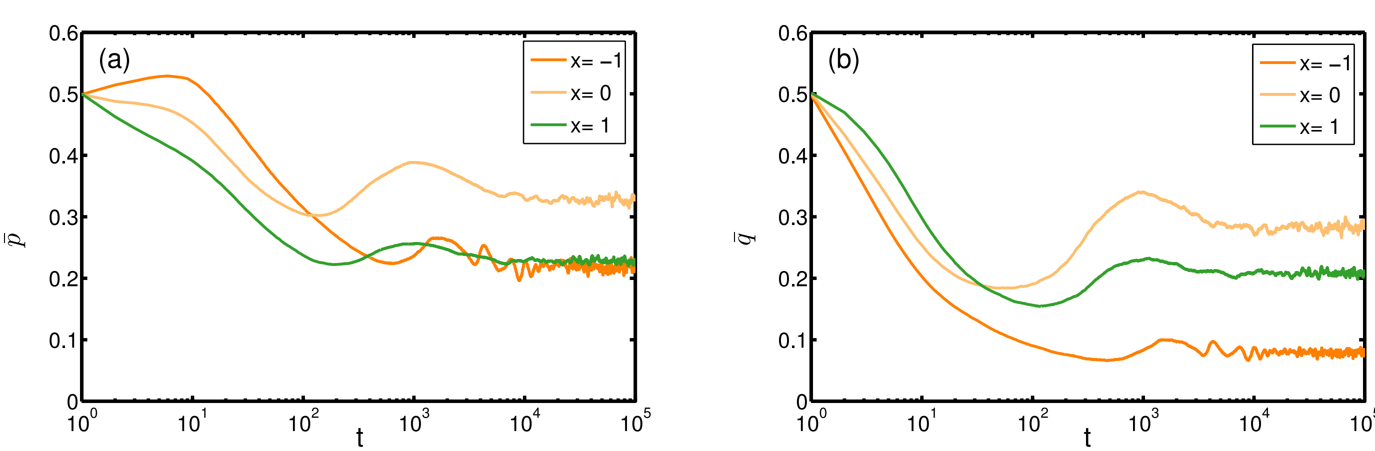

Fig. 2. (Color on-line) Time evolution of $\bar{p}$ (Fig. 2a) and $\bar{q}$ (Fig. 2b) with different x. Parameters: MCS = 10^5, N = 100*100, ε = 0.001.

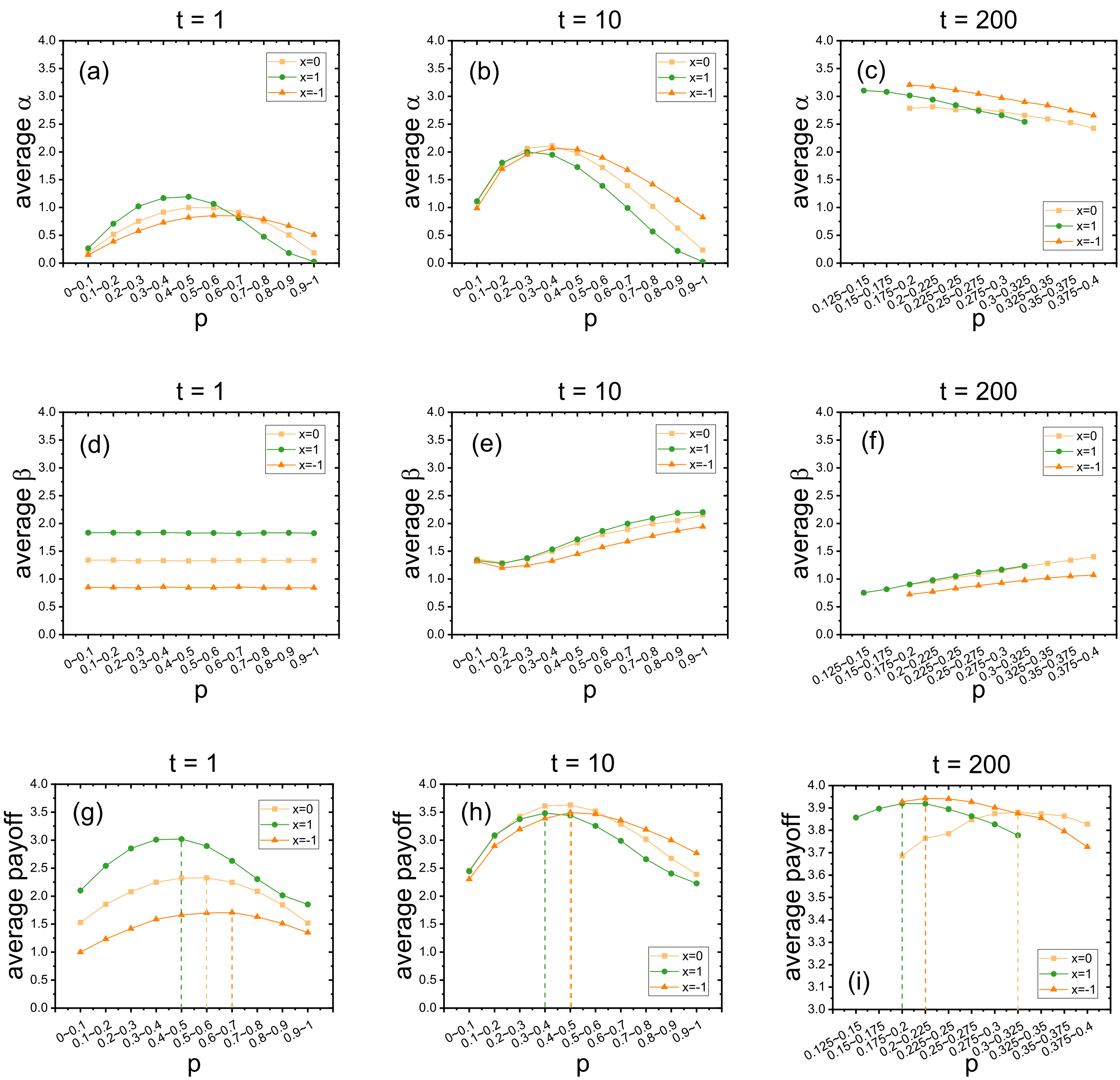

Fig. 3. (Color on-line) Average α, average β and average payoff ($\alpha + \beta$) of the nodes according to p values for x = -1, x = 0 and x = 1 at time steps t = 1, t = 10 and t = 200. The $\bar{p}$ of nodes that have the highest payoff ($p_{\max}$) at different time steps for different x are marked with dashed lines in Fig. 3(g-i). Parameters: N = 100*100, ε = 0.001.

To understand the evolution process toward such a result, we investigate the time evolution of $\bar{p}$ and $\bar{q}$ for different x. Fig. 2a and 2b show the time evolution of $\bar{p}$ and $\bar{q}$ of the system for $x = 1$, $x = 0$ and $x = -1$, respectively. According to Fig. 2a, $\bar{p}$ of $x = 1$ is lower than standard ($x = 0$) across the whole evolution time, while $\bar{p}$ of $x = -1$ go through a short increase at the first 10 generations and then decreases quickly to a level close to the $x = 1$ condition. From Fig. 2b, we can see that $\bar{q}$ of $x = 1$ decreases slower than standard in the first few generations but faster in the later generations. However, $\bar{q}$ of $x = -1$ decreases faster than the standard from the start to the end.

Later, to further uncover the effect of similarity, we analyze the payoff distribution at different time steps of the evolution in Fig. 3. We divide the payoff into two parts: the payoff received as proposer α and the payoff received as responder β. That is,

$$P_i = \alpha_i + \beta_i = (1 - p_i') \cdot n + \sum_{k \in \aleph_i} p_k', \tag{5}$$

where n denotes the number of successful collaborations when player i acts as proposer, and $\aleph_i$ represents the neighborhood whose offers are accepted by player i when it acts as responder, with p_k' being the money received from each one. According to the evolution rule, strategies that obtain higher payoffs will have more opportunities to be inherited by the next generation. Therefore, by observing the $\bar{p}$ of nodes that have the highest payoff ($p_{\max}$) at different time steps, we can understand how $\bar{p}$ evolves as the evolution proceeds.

Here, we discuss the two conditions of $x = 1$ and $x = -1$ separately. For $x = 1$, the population tends to propose higher p and hold lower q to other players. Hence, the offer between players is more likely to be accepted. At the start of the evolution, with the increase of n and the decrease of $(1 - p')$, the variation of $\alpha = n \cdot (1 - p_i')$ becomes theoretically uncertain. However, we can see from Fig. 3a that the peak of α moves slightly left compared to $x = 0$. From Fig. 3d, β follows a uniform distribution because it is determined by only the p of a player's neighbors, not the p of the player. Take the sum of α and β, that is, the overall payoff. $p_{\max}$ of $x = 1$ is smaller than that of $x = 0$ (Fig. 3g), which means that a smaller p takes an advantageous position with the effect of similarity at the start of the evolution. This phenomenon can also be seen in the 10th generation (Fig. 3h) and the 200th generation (Fig. 3i), where $p_{\max}$ of $x = 1$ is always smaller than that of $x = 0$ (for detailed information, please see Fig. 3b, c, e, and f). Naturally, the smaller $p_{\max}$ leads the population to a lower $\bar{p}$ level than the standard. As x increases, the effect of similarity also increases. The $p_{\max}$ of the system consequently decreases, which results in a decrease in the $\bar{p}$ of the system with an increase in x.

For $x = -1$, the population tends to propose lower p and hold higher q to other players. Hence, the offer between players is more likely to be rejected. As we can see from Fig. 3g, $p_{\max}$ of $x = -1$ is larger than that of $x = 0$, which means that higher p takes an advantageous position relying on more successful collaborations at the start of the evolution. However, with similarity adding an incremental effect to the q of the population, only very small q can survive the first few generations and become dominant (See Fig. 2b, average q decreases fastest when $x = -1$). Proposers would have no necessity to propose high offers in this case. Therefore, compared with the condition of $x = 0$, the $p_{\max}$ of $x = -1$ not only decreases but also decreases faster. By the 10th generation, $p_{\max}$ of $x = -1$ becomes nearly equal to that of $x = 0$ (Fig. 3h); by the 200th generation, $p_{\max}$ is already smaller than that of $x = 0$ (Fig. 3i). Thus, this advantage of a smaller $p_{\max}$ leads the population to an equilibrium with a smaller $\bar{p}$ than the standard (Fig 2a). As x decreases, the effect of similarity increases. The $p_{\max}$ of the system consequently decreases. This makes the $\bar{p}$ decrease with the decrement of x.

To further illustrate the depressing effect of similarity on fairness, we employ a typical toy model to provide a quantitatively observable analysis. Fig. 4a shows a snapshot of a standard UG model. The specific (p, q) value of each node is illustrated in the figure. Here, we focus on four clustering nodes: A, B, C and D. Compared with the mean value of p and q in the population, we denote the

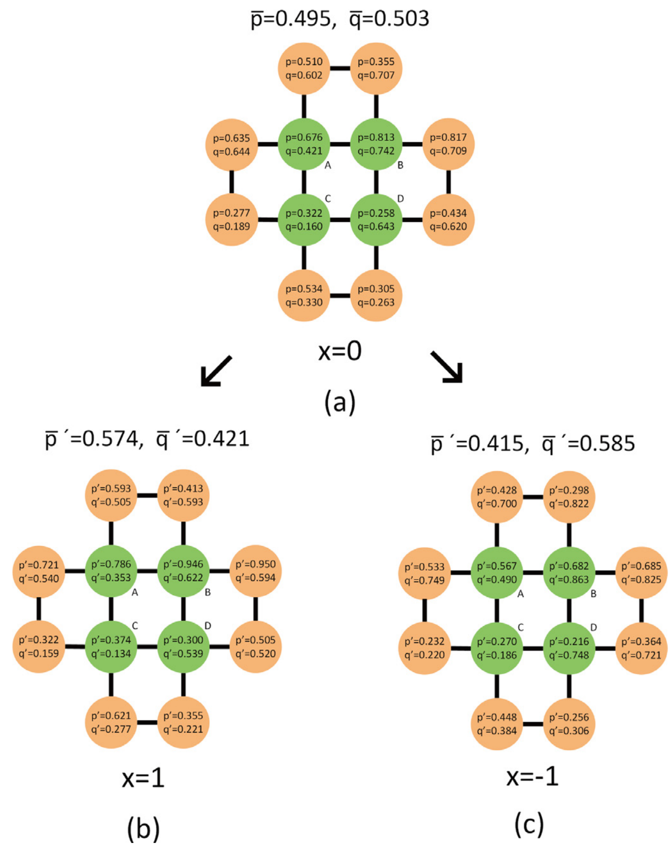

Fig. 4. A toy model of an evolutionary ultimatum game in a lattice network. Three cases of x = 0, x = -1 and x = 1 are shown with specific strategy values.

strategies of A, B, C and D as H(*high*) and L(*low*). If the strategy is higher than average, we denote it as H; otherwise, we denote it as L. Accordingly, we obtain A (H, L), B (H, H), C (L, L) and D (L, H). Similarity is set to a fixed value, which is the statistical average of a real network, to simplify the calculation. The specific values of p' and q' in the above two situations are shown in Fig. 4b and 4c, and the corresponding payoffs of the four nodes are shown in Table 1.

Consider the content of Table 1. Compared with the standard UG (x = 0), the number of successful splits increases when x = 1 and decreases when x = -1. For total payoff, we overstrike the

Table 1
The specific number of successful splits and payoffs for nodes A, B, C and D after one round of the game in the toy model when x=0, 1 and -1. m denotes the number of successful splits. The outcome indicates that the strategy with lower p and lower q (strategy C) can obtain relatively higher payoffs than other strategies in the presence of similarity ($x = \pm 1$).

x			A (H,L)	B (H,H)	C (L,L)	D (L,H)
0	proposer	m	3	4	1	1
		payoff	0.972	0.784	0.678	0.742
	responder	m	3	1	4	1
		payoff	1.958	0.817	1.745	0.813
	total payoff		**2.930**	1.601	2.423	1.555
1	proposer	m	4	4	3	2
		payoff	0.856	0.216	1.878	1.400
	responder	m	4	2	4	1
		payoff	2.634	1.736	2.029	0.946
	total payoff		3.490	1.952	**3.907**	2.346
-	proposer	m	1	1	1	1
1		payoff	0.433	0.318	0.730	0.784
	responder	m	2	0	4	0
		payoff	1.215	0	1.463	0
	total payoff		1.648	0.636	**2.193**	0.784

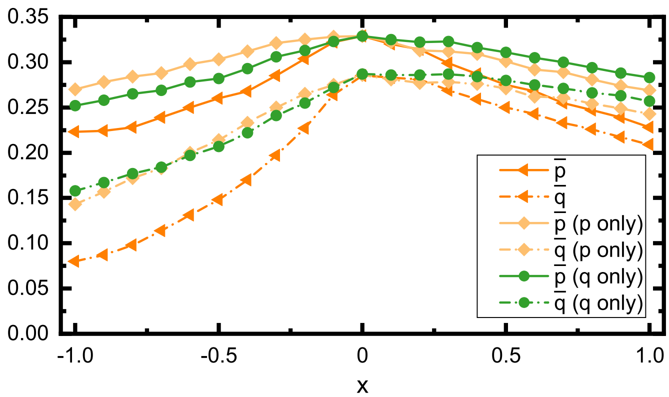

Fig. 5. (Color on-line) $\bar{p}$ and $\bar{q}$ as a function of x with similarity taking effect on both p and q, p only and q only. Parameters: MCS = 10^6, N = 100*100, ε = 0.001.

highest splits to emphasize its advantageous position. We can see that smaller p and smaller q (strategy C) have been exemplified to obtain relatively higher payoffs in the presence of similarity, both when $x = 1$ and $x = -1$.

In former discussions, similarity is set to have an effect on both p and q. Here, we separate the effect of this factor on p and q independently. We conduct simulations with similarity affecting ① both p and q ② p only and ③ q only. As we can see from Fig. 5, the variation in $\bar{p}$ and $\bar{q}$ under all three circumstances follow a similar trend of peaking at $x = 0$ and decline as $|x|$ increases. However, the average $\bar{p}$ and $\bar{q}$ values of the solely affecting condition (yellow and green line) are higher than those of both affecting conditions. That is, the suppressing effect of similarity on fairness does not disappear because of the lack of one affecting object, but it weakens to some extent, which makes the result slightly closer to fairness.

In the following experiments, we demonstrate the $\bar{p}$ and $\bar{q}$ under different distributions of s (note that similarity Δs is defined as $|s_i - s_j|$). As shown in Fig. 6, the variation in p and q basically follow the same trend as the distribution changes. Regardless of whether s follows a uniform distribution, a Gaussian distribution $(f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \mu = 0.5, \sigma^2 = 0.02, 0 < x < 1)$, an exponential distribution $(f(x) = \lambda e^{-\lambda x}, \lambda = 5, 0 < x < 1)$ or a power-law distribution $(f(x) = cx^{-\alpha-1}, \alpha = 2, c = \frac{1}{5000}, 0 < x < 1)$, similarity plays a suppressing role in the fairness of the system, which proves that the effect of similarity over fairness in an evolutionary UG is universal.

The mutation rate used in our research is a maintained constant: $\varepsilon = 0.001$, which is a commonly used value in UG studies. However, the mutation rate ε has been proved of having an effect on the cooperation in PDG [49] and the fairness of UG [9,20,28]. To illustrate this effect under the specific presence of similarity, here we display the $\bar{p}$ with different ε when $x = 0$, -1 and 1, in Table 2. As we can see, with all different mutation rate, similarity causes a decrease in the fairness of the system both when $x = -1$ and 1, which supports our conclusion. Furthermore, we also validate the

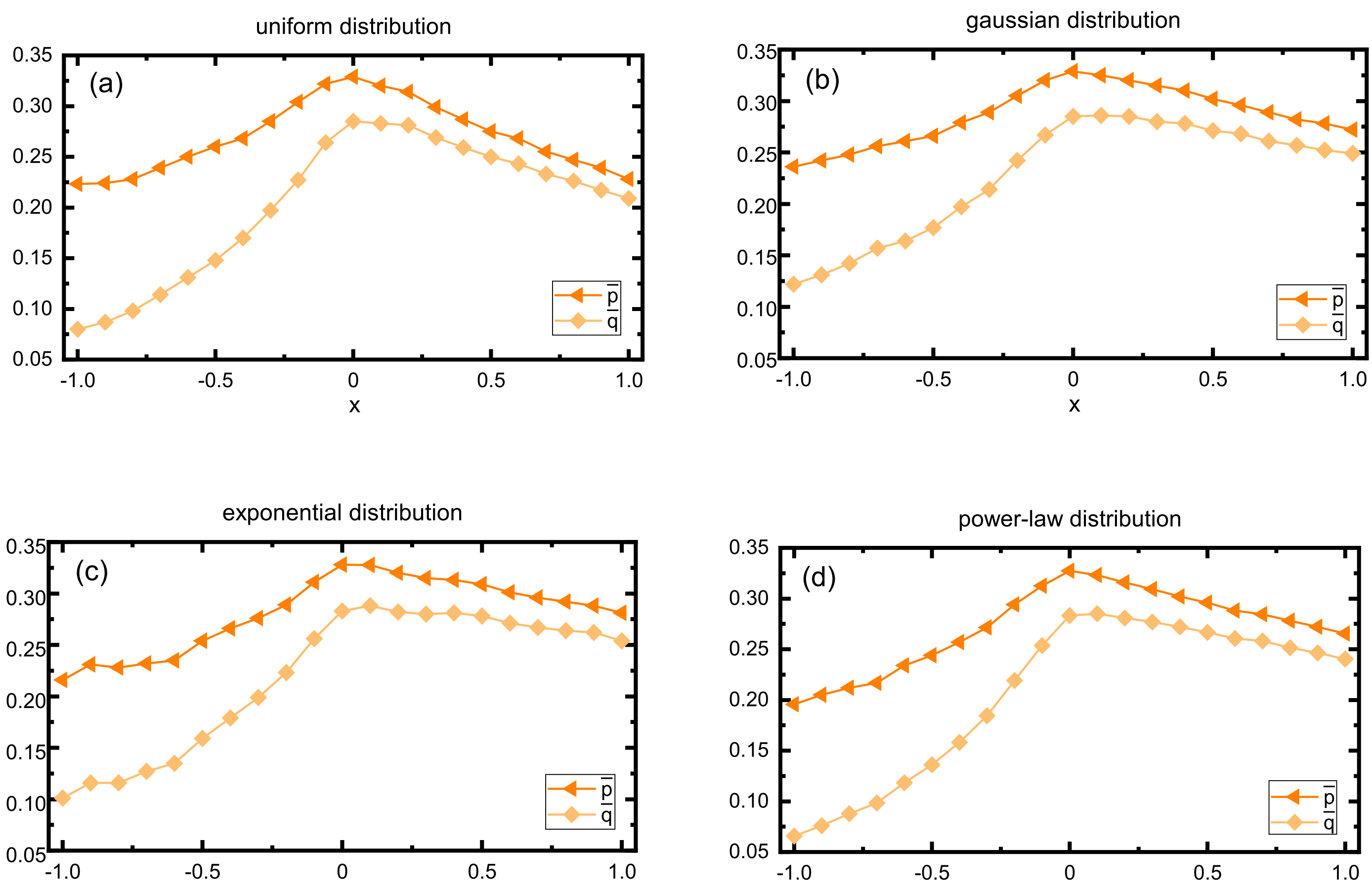

Fig. 6. (Color on-line) $\bar{p}$ and $\bar{q}$ as a function of x when s follows a uniform distribution (Fig. 6a), a Gaussian distribution (Fig. 6b), an exponential distribution (Fig. 6c) and a power-law distribution (Fig. 6d). It can be observed in the figure that all four distributions have a similar effect on the decrement of fairness among the network. Parameters: MCS = 10^6, N = 100*100, ε = 0.001.

Table 2
$\bar{p}$ with various mutation rate ε for x = 0, x = 1 and x=-1. Parameters: MCS = 10^6, N = 100*100.

ε	x=-1	x=0	x=1
0.01	0.218	0.239	0.171
0.005	0.215	0.277	0.190
0.002	0.223	0.317	0.216
0.001	0.223	0.329	0.228

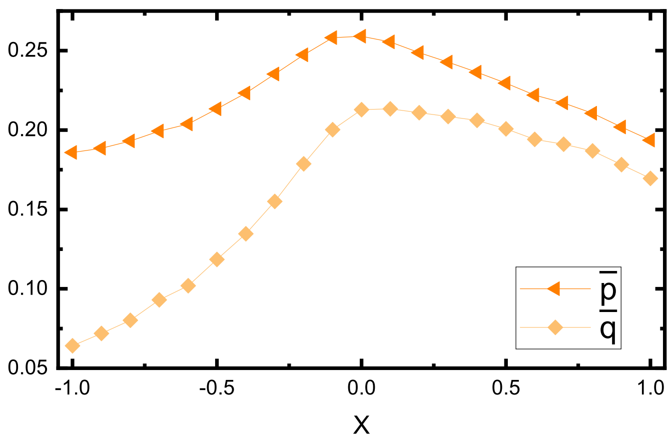

Fig. 7. (Color on-line) Average offer ($\bar{p}$) and acceptance threshold ($\bar{q}$) as a function of x in a Moore lattice network of k=8. Parameters: MCS = 10^6, N = 100*100, ε = 0.001.

robustness of our result in a Moore lattice network with k=8. The result, as shown in Fig. 7, supports our conclusion as well.

4. Conclusion

In this paper, we studied how similarity between individuals affects the evolution of fairness in a spatial ultimatum game (UG). In our model, the offer (p) and the acceptance threshold (q) in a game were affected by the similarity between the two players. With different values of variate x, one could either be more generous and tolerant to other players ($x > 0$) or stingier and stricter ($x < 0$). According to our simulation result, in both situations of $x > 0$ and $x < 0$, the fairness of the system decreased with the effect of similarity. Equal treatment regardless of similarity ($x = 0$) was the best strategy to obtain fairness. To explain this phenomenon, we presented the time evolution of $\bar{p}$ and $\bar{q}$, followed by a detailed analysis of the payoff distribution. This explanation was further supported by a typical toy model, which provided a quantitative illustration. Moreover, we proved that the depressing effect of similarity on fairness not only exists when p and q were added simultaneously but also when p and q were affected independently. Furthermore, the effects of different distributions of similarity and mutation rates were also discussed in this paper. We believe that our work will explain the studies of fairness among human societies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant 61671031, Grant 61722102, and Grant 91738301.

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